Exercise 141

For the following exercises, simplify each expression by writing it in terms of sines and cosines, then simplify. The final answer does not have to be in terms of sine and cosine only.

$$\frac{\tan^2 x}{\sec^2 x}$$

Solution

One way is by

$$\begin{aligned}\frac{\tan^2 x}{\sec^2 x} &= \frac{\sec^2 x - 1}{\sec^2 x} \\ &= \frac{\sec^2 x}{\sec^2 x} - \frac{1}{\sec^2 x} \\ &= 1 - \cos^2 x \\ &= \sin^2 x.\end{aligned}$$

Another way is by

$$\begin{aligned}\frac{\tan^2 x}{\sec^2 x} &= \tan^2 x \cos^2 x \\ &= \left(\frac{\sin^2 x}{\cos^2 x}\right)\cos^2 x \\ &= \sin^2 x.\end{aligned}$$